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Two-state vector formalism and quantum interference

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Abstract

We show that two-state vector formalism (TSVF), applied to quantum systems that make use of delicate interference effects, can lead to paradoxes. We consider a few schemes of nested Mach–Zehnder interferometers that make use of destructive interference. A particular interpretation of TSVF applied to these schemes makes predictions that are contradictory to quantum theory and can not always be verified. Our results suggest that TSVF might not be a suitable tool to describe quantum systems that make use of delicate quantum interference effects.

Keywords: two-state vector formalism, weak measurements, quantum interference

(Some figures may appear in colour only in the online journal)

Two-state vector formalism (TSVF) is a relatively new tool for studying quantum systems and has emerged as a consequence of exploring time symmetry in quantum mechanics [1]. The idea is that during the time between two strong projective measurements (pre- and post-selection) the quantum system is described by not one but two state vectors; one evolving forward in time, and the other that starts at the time of the second measurement and evolves backward in time [2, 3]. The formalism uses the information from these two state vectors, and makes predictions for the outcome of certain measurements on the system during the time between pre- and the post-selection. These predictions can be verified by making weak measurements [4–6] on the systems. The ability to realize such measurements has attracted significant interest in recent years, and the formalism has been used to explore a lot of exciting physics [7–15]. Other more general formulations to study pre- and post-selected quantum systems have also emerged [16–22].

A subtle point regarding the use of TSVF concerns the back-action of the weak measurement on the quantum system. It is generally assumed, and has been demonstrated in a large number of studies [7–15], that the weak measurements (because of their nature) do not significantly disturb the quantum system. However this assumption fails down for the quantum systems that make use of delicate destructive interference effects, as any disturbance to destructive interference is always significant. Thus, not surprisingly, the use of TSVF on quantum systems that make use of destructive interference has given rise to a number of controversies [23–32]. In this paper we address one such controversy, which is related to the path information of a quantum particle in a nested Mach–Zehnder interferometer [23–25, 29]. The resolution of this controversy has profound implications for our fundamental understanding of basic quantum mechanics. It is to be noted that this particular shortcoming of TSVF is avoided in more general formulations [16–22] by taking into account the effects of weak measurements at all orders.

The paper is organized as follows. In section 1 we give a summary of a recent theory of the past of quantum particle based on TSVF and discuss the paradox the theory gives rise to. In section 2 we show that the paradox does not arise in standard quantum mechanics. In section 3 we apply the theory to a scheme where multiple weak measurements are performed. We show that the quantum description of the system that takes into account the higher order effects (with respect to the weak measurement perturbation) can be different from the prediction of the theory [23]. In section 4 we consider a system in which the destructive interference is modified by a weak measurement in a manner that is very different from the one considered implicitly in [23]. In this system the weak measurement can restore destructive interference along a channel that already had a tiny leakage. For such a setup the prediction of the theory of the past of the particle is in clear contradiction with standard quantum mechanics. Finally we conclude in section 5.

1. The past of a quantum particle—a paradox

Consider a nested Mach–Zehnder interferometer as shown in figure 1(a). The outer bigger interferometer has beam splitters BS_1 of reflectivity r and transmissivity t (both assumed to be real). The inner interferometer along arm B of the outer interferometer has 50:50 beam splitters. This makes the output port F of the inner interferometer (the one that leads to the detector D) a dark port, and thus any wave packet entering the inner interferometer can not reach the detector D. Similarly a wave packet coming from the detector D can not reach the source S through the inner interferometer. Now consider a particle coming from the source S that has been detected (post-selected) at D. Simple quantum mechanical calculations show that the probability amplitude of this event is r^2, and this shows clearly that the particle has followed the arm A of the outer interferometer. Indeed, if the particle had come from inside the inner interferometer the post-selection probability would have been modified with a term $\propto t^2$, signifying that the particle has to be transmitted through the beam-splitters BS_1 twice on its way from the source S to the detector D. Thus for the given pre-selection (particle coming

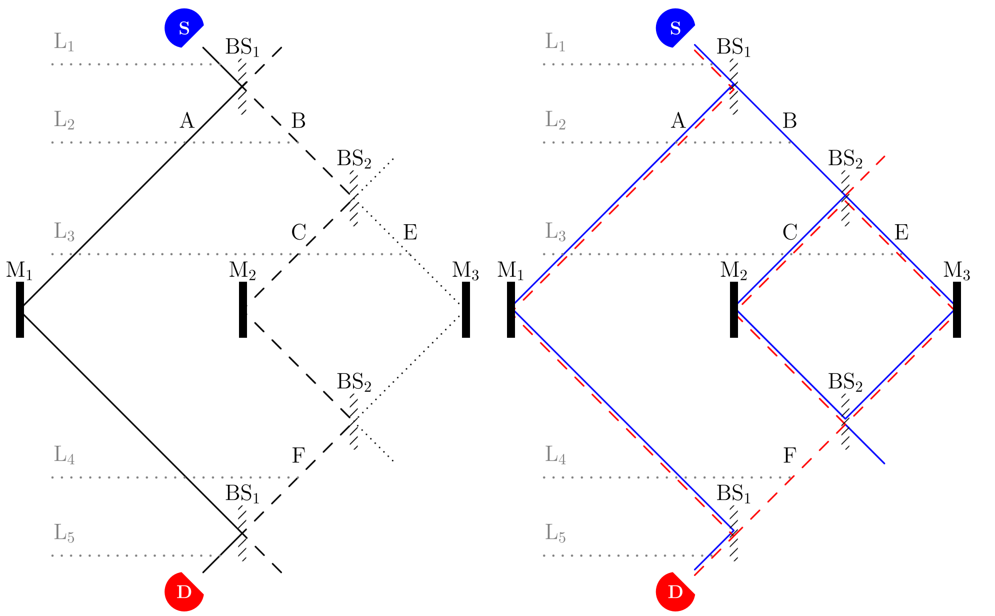

Figure 1. (a) A scheme with nested Mach–Zehnder interferometers. The solid, dashed, and dotted lines show three quantum modes (channels) for a photon between the source S and the detector D. (b) System in TSVF with the forward (solid blue) and the backward (dashed red) evolving states.

Table 1. Forward and backward evolving states for the system in figure 1(b). The forward evolving state is represented by the column vector $(n_1\ n_2\ n_3)^\dagger$ where n_1, n_2, and n_3 represents the number of photons along the three quantum channels in figure 1(a). The backward evolving states are represented by row vectors. The table also shows the weak values defined as $\langle \hat{n}_i \rangle_{\rm w} = \langle \phi | \hat{n}_i | \psi \rangle / \langle \phi | \psi \rangle$ for the photon number operators $\hat{n}_i$ for the corresponding modes.

Stage	Forward evolving $\|\psi\rangle$	Backward evolving $\langle\phi\|$	Weak values		
			$\langle \hat{n}_1 \rangle_{\rm w}$	$\langle \hat{n}_2 \rangle_{\rm w}$	$\langle \hat{n}_3 \rangle_{\rm w}$
L1	$(1\quad 0\quad 0)^\dagger$	$(-r^2\quad {\rm i}rt\quad {\rm i}t)$	1	0	0
L2	$(-{\rm i}r\quad t\quad 0)^\dagger$	$({\rm i}r\quad 0\quad {\rm i}t)$	1	0	0
L3	$(-{\rm i}r\quad -{\rm i}t/\sqrt{2}\quad t\sqrt{2})^\dagger$	$({\rm i}r\quad {\rm i}t/\sqrt{2}\quad t/\sqrt{2})$	1	$t^2/2r^2$	$-t^2/2r^2$
L4	$(-{\rm i}r\quad 0\quad -{\rm i}t)^\dagger$	$({\rm i}r\quad t\quad 0)$	1	0	0
L5	$(-r^2\quad -{\rm i}rt\quad -{\rm i}t)^\dagger$	$(1\quad 0\quad 0)$	1	0	0

from S) and the post-selection (particle detected at D), the path of the particle has to be associated with arm A of the outer interferometer.

The straightforward quantum mechanical reasoning discussed above has been challenged in a series of papers [23, 26, 30], and it has been claimed that the particle on its way from the source S to the detector D has been inside the inner interferometer. The reason for this claim is the overlap of the state of the particle with the backward propagating state (in TSVF) that starts from the detector D at the time of post-selection and runs backward in time towards the source S. These forward and backward propagating states are given in table 1 and are shown

Table 2. Quantum evolution of the system with a weak measurement (measurement strength parameter ϵ) inside the inner interferometer along the arm E. The crucial point is the mode $(0\ 1\ 0)^\dagger$ at stage L4, which is no longer empty and thus contributes to the post-selection at the detector.

Stage	Quantum State $\|\psi\rangle$
L1	$(1 \quad 0 \quad 0)^\dagger$
L2	$(-\mathrm{i}r \quad t \quad 0)^\dagger$
L3	$(-\mathrm{i}r \quad -\mathrm{i}t/\sqrt{2} \quad t(1+\mathrm{i}\epsilon)/\sqrt{2})^\dagger$
L4	$(-\mathrm{i}r \quad \mathrm{i}\epsilon t/2 \quad t(-\mathrm{i}+\epsilon/2))^\dagger$
L5	$((-r^2+\mathrm{i}\epsilon t^2/2) \quad rt(-\mathrm{i}+\epsilon/2) \quad t(-\mathrm{i}+\epsilon/2))^\dagger$

in figure 1(b). The proposed theory [23] relates the past of the particle between the pre and the post-selection with the overlap of the forward and the backward evolving waves. Another measure to describe the past behavior of the particle is the weak value [33] of some suitable operator e.g. the photon number operator along different arms of the nested interferometer. These are also given in table 1. The theory states that the particle during its past (between pre- and post-selection) was present where the forward and the backward evolving waves overlap and the weak value of some suitable operator (e.g. the photon number operator) is non-zero. This means that the particle was present both along arm A of the outer interferometer and inside the inner interferometer, but was not on the paths that lead to and come out of the inner interferometer. The theory thus gives rise to the paradox that the photon did not enter the inner interferometer, did not come out of the inner interferometer, but was inside the inner interferometer [26].

2. Resolution of the paradox

The paradox discussed in the previous section has been claimed in a recent experiment [30] using weak measurements. However the paradox appears only in the particular interpretation of TSVF [23, 30] and in standard quantum mechanics there is no paradox [24]. The detailed quantum mechanical calculation of a weak measurement inside the inner interferometer of the setup shown in figure 1(a) has been carried out [24], and it has been shown that the quantum evolution of the system subsequent to a weak measurement inside the inner interferometer is different from the evolution when no measurement is performed. When a weak measurement is performed along one arm of the inner interferometer, it disturbs the destructive interference along the dark port of the inner interferometer and a tiny amplitude leaks towards the detector D. For the weak measurement with the measurement strength ϵ, this leakage is at first order in ϵ. This leaked amplitude combines with the amplitude along the arm A of the outer interferometer and contributes to the post-selection. This can be seen in table 2 where standard forward evolving states of the system are given (with weak measurement along the arm E of the inner interferometer at stage L3). If the weak measurement is not performed inside the inner interferometer then the destructive interference along the arm F is not disturbed. This leaves only one quantum channel, i.e. along arm A of the outer interferometer, for the particle to reach the detector. This also explains why a weak measurement on the path leading to, and the path coming out of the inner interferometer can not reveal the presence of the photon. If a weak measurement is performed along the arm F subsequent to the weak measurement inside the inner interferometer then the particle will be revealed along the arm F at the lowest order of the measurement strength parameter.

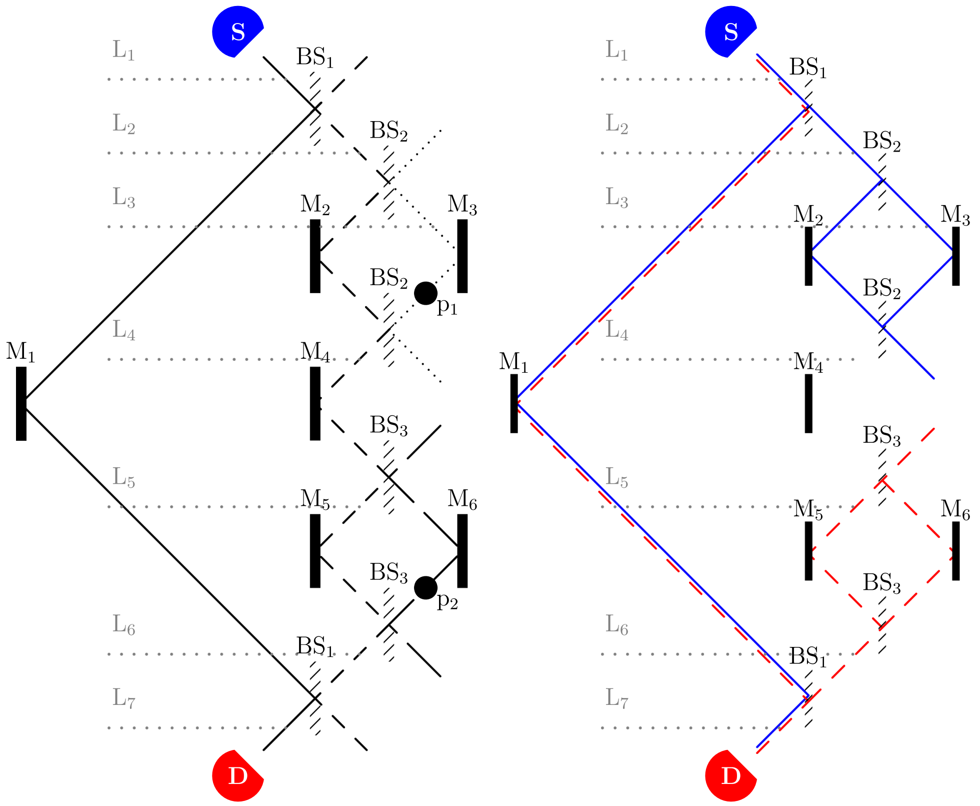

Figure 2. (a) Another scheme with nested Mach–Zehnder interferometers. The solid, dashed, dotted, and long dashed lines designate four quantum modes that can describe the state of the photon between the source S and the detector D. (b) System in TSVF with the forward (solid blue) and the backward (dashed red) evolving states.

It must be noted that the leakage along arm F subsequent to the weak measurement inside the inner interferometer is also very important for the proposed theory [23], and has been considered implicitly in the theory [23, 25]. Without this leakage the prediction of the theory for the presence of the particle inside the inner interferometer cannot be verified with standard quantum mechanics.

3. TSVF with two weak measurements

In the last section it was shown that the prediction of the theory [23] based on TSVF for the presence of the particle inside the inner interferometer in the setup in figure 1(a) is correct only if the system is slightly disturbed by performing a weak measurement on the system. In this section we will show a setup in which the prediction of the theory for the weak measurements are correct only if the measurements are not performed.

Consider the nested interferometer scheme shown in figure 2(a), which consists of an outer bigger interferometer MZ_1 and two inner interferometers MZ_2 and MZ_3 placed along the right arm of the outer interferometer. The outer interferometer has beam splitters BS_1 of reflectivity r and transmissivity t, which are both assumed to be real. All the other beam

Table 3. Forward evolving state in quantum mechanics and the backward evolving state of TSVF.

Stage	Forward evolving $\vert\psi\rangle$	Backward evolving $\langle\phi\vert$
L1	$(1 \quad 0 \quad 0 \quad 0)^{\dagger}$	$(r^2 \quad \mathrm{i}rt \quad 0 \quad \mathrm{i}t)$
L2	$(-\mathrm{i}r \quad t \quad 0 \quad 0)^{\dagger}$	$(\mathrm{i}r \quad 0 \quad 0 \quad \mathrm{i}t)$
L3	$(-\mathrm{i}r \quad -\mathrm{i}t/\sqrt{2} \quad t\sqrt{2} \quad 0)^{\dagger}$	$(\mathrm{i}r \quad 0 \quad 0 \quad \mathrm{i}t)$
L4	$(-\mathrm{i}r \quad 0 \quad -\mathrm{i}t \quad 0)^{\dagger}$	$(\mathrm{i}r \quad 0 \quad 0 \quad \mathrm{i}t)$
L5	$(-\mathrm{i}r \quad 0 \quad -\mathrm{i}t \quad 0)^{\dagger}$	$(\mathrm{i}r \quad \mathrm{i}t/\sqrt{2} \quad 0 \quad t\sqrt{2})$
L6	$(-\mathrm{i}r \quad 0 \quad -\mathrm{i}t \quad 0)^{\dagger}$	$(\mathrm{i}r \quad t \quad 0 \quad 0)$
L7	$(-r^2 \quad -\mathrm{i}rt \quad -\mathrm{i}t \quad 0)^{\dagger}$	$(1 \quad 0 \quad 0 \quad 0)$

splitters, BS_2 for MZ_2 and BS_3 for MZ_3 are 50:50 beam splitters. This arrangement means that the mirror M_4 is the dark port of MZ_2 for the photon coming from the source S, and it is also the dark port for MZ_3 for a quantum wave coming from the detector D. The right sides of the output ports of MZ_2 and MZ_3 are leakage points from where the photon coming from the source can leave the system. L1, L2, etc are various stages during the quantum evolution of the system.

We consider a single photon coming from the source S that has been detected at the detector D and use column vectors to describe the quantum state of the photon. The basis vector $(1\ 0\ 0\ 0)^{\dagger}$ represents the mode in which the photon coming from the source is reflected by beam splitters BS_1. The mode $(0\ 1\ 0\ 0)^{\dagger}$ is the one in which the photon coming from the source is transmitted through the first beam splitter BS_1 and is reflected by each beam splitter on its subsequent path. Finally the modes $(0\ 0\ 1\ 0)^{\dagger}$ and $(0\ 0\ 0\ 1)^{\dagger}$ represent states in which the photon remains on the RHS of BS_2 and BS_3, respectively. These four modes are shown by solid, dashed, dotted, and long dashed lines respectively in figure 2(a). We also define the photon creation (annihilation) operators $\hat{a}_n^{\dagger}\,(\hat{a}_n)$ for the last two modes through the relations $[(0\,0\,0\,0)\hat{a}_3]^{\dagger} = (0\,0\,1\,0)^{\dagger}$ and $[(0\,0\,0\,0)\hat{a}_4]^{\dagger} = (0\,0\,0\,1)^{\dagger}$. The transformation matrices for the beam splitters BS_1, BS_2, and BS_3 are Q_1, Q_2, and Q_3, respectively, and are given as

$$Q_1 = \begin{pmatrix} ir & t & 0 & 0 \\ t & ir & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix};\ Q_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & i & 1 & 0 \\ 0 & 1 & i & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{pmatrix};\ Q_3 = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & i & 0 & 1 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 1 & 0 & i \end{pmatrix}. \tag{1}$$

The standard forward quantum evolution of the single photon coming from the source S is given in table 3. At stage L3 the photon is present both along the left arm of the outer interferometer and inside the first inner interferometer MZ_2. However, the amplitude from inside the inner interferometer MZ_2 can not continue along the mode $(0\ 1\ 0\ 0)^{\dagger}$ because of the destructive interference towards the mirror M_4 and leaks out of the system at stage L4. Accordingly, the probability of the detection of the photon at the detector D is r^4 and it shows that the photon has passed through the left arm of the outer interferometer. The prediction from TSVF discussed in the previous section also agrees with this description. The backward evolving state is also shown in table 3 and in figure 2(b). This backward evolving state coming from the detector is present inside the second inner interferometer MZ_3 and it also leaves the system at stage L4 because of of the destructive interference towards the mirror M_4. Consequently the overlap of the forward and the backward evolving states is only along the left arm of the outer interferometer and the weak value of the photon number operator is non-zero only for the mode $(1\ 0\ 0\ 0)^{\dagger}$. Thus we see that the prediction from TSVF is in accordance with the standard quantum theory. However unlike the system [23] where the prediction for the photon inside the inner interferometer was true only if the weak measurement for the presence of the photon was actually performed, in this system, the prediction run into problem if the weak measurements are performed on the system. For one weak measurement in either inner interferometer, the prediction of TSVF remains valid, bur for two weak measurements —one in each inner interferometer—the prediction can be violated. We next consider weak measurements on the system in figure 2(a) for revealing the presence/trace of the photon inside the inner interferometers MZ_2 and MZ_3.

The trace of the photon along the right arm of the outer interferometer can be investigated through weak measurements using a number of methods, such as propagation in Kerr medium [11], propagation through titled glass plates [34], reflecting the photon from vibrating mirrors [30], or through dispersive coupling with atoms [24]. We follow the latter scheme and consider the following Hamiltonians to carry out two weak quantum non-demolition measurements (taking $\hbar = 1$)

$$\hat{H}_1 = \epsilon_1 \hat{a}_3^\dagger a_3 \ |b_1\rangle\langle b_1| \ \delta(t - t_1), \tag{2a}$$

$$\hat{H}_2 = \epsilon_2 \hat{a}_4^\dagger a_4 \ |b_2\rangle\langle b_2| \ \delta(t - t_2). \tag{2b}$$

Here ϵ_i represent the measurement interaction strength, and $|b_i\rangle\langle b_i|$ are the operators for the two measuring devices. The first device in the first inner interferometer MZ_2 makes a weak measurement for the photon in the mode $(0\ 0\ 1\ 0)^\dagger$ at time t_1. The second measurement by $\hat{H}_2$ is carried out in the second interferometer MZ_3 for the photon in the mode $(0\ 0\ 0\ 1)^\dagger$. Such weak measurements can be realized by the dispersive coupling of the photon on upper transitions in the three-level atoms in the ladder configuration [24, 35, 36]. We use two such atoms, one in each inner interferometer at locations p_1 and p_2, respectively. Each atom is initially prepared in the superposition of the two lower states,

$$|A\rangle = \frac{|b_1\rangle + |c_1\rangle}{\sqrt{2}}; \ |B\rangle = \frac{|b_2\rangle + |c_2\rangle}{\sqrt{2}} \tag{3}$$

and couples to the respective photon mode. If the concerned modes are occupied then the atoms evolve into

$$|A'\rangle = \frac{e^{-i\epsilon_1} \ |b_1\rangle + |c_1\rangle}{\sqrt{2}}; \ |B'\rangle = \frac{e^{-i\epsilon_2} \ |b_2\rangle + |c_2\rangle}{\sqrt{2}}, \tag{4}$$

under the action of the measurement Hamiltonian (2). The atoms can be probed, conditional to the post-selection, to investigate the trace of the photon in the arm B. A single run of the experiment will not give much information. However, if the experiment is performed a large number of times then the probabilities $P(k_1, l_2)$ of finding the atom 1 in the state $|k\rangle$ and the atom 2 in the state $|l\rangle$ will reveal the trace of the photon.

We next consider the evolution of the photon and the atoms through different quantum channels with initially atom 1 in state $|A\rangle$, atom 2 in state $|B\rangle$, and the photon coming from the source in the state $(1\ 0\ 0\ 0)^\dagger$. The combined state of the system and the probe through successive stages is

$$|\psi_{L2}\rangle = (-\mathrm{i}r \quad t \quad 0 \quad 0)^\dagger |A\rangle|B\rangle, \tag{5a}$$

$$\begin{aligned} |\psi_{L4}\rangle = &-(\mathrm{i}r \quad t/2 \quad \mathrm{i}t/2 \quad 0)^\dagger |A\rangle|B\rangle \\ &+ (0 \quad t/2 \quad -\mathrm{i}t/2 \quad 0)^\dagger |A'\rangle|B\rangle, \end{aligned} \tag{5b}$$

$$|\psi_{L6}\rangle = (-\,\mathrm{i}r \quad t/4 \quad -\mathrm{i}t/2 \quad \mathrm{i}t/4)^{\dagger}|A\rangle|B\rangle \\ +(0 \quad -t/4 \quad 0 \quad \mathrm{i}t/4)^{\dagger}|A\rangle|B'\rangle \\ +(0 \quad -t/4 \quad -\mathrm{i}t/2 \quad -\mathrm{i}t/4)^{\dagger}|A'\rangle|B\rangle \\ +(0 \quad t/4 \quad 0 \quad -\mathrm{i}t/4)^{\dagger}|A'\rangle|B'\rangle. \tag{5c}$$

We can see that the photon wave packet at the stages L4 and L6 is different from the pre-selected state shown in figure 2(b) and table 3. The crucial difference is the non-vanishing amplitude along the mode $(0\ 1\ 0\ 0)^{\dagger}$ at the stage L6. In TSVF this mode is assumed to be empty. The assumption is correct for no measurement or for only a single weak measurement. However, with two weak measurements performed by the operations $|\psi_{L4}\rangle = Q_2 \mathrm{e}^{-\mathrm{i}\int \hat{H}_1 \mathrm{d}t} Q_2\ |\psi_{L2}\rangle$ and $|\psi_{L6}\rangle = Q_3 \mathrm{e}^{-\mathrm{i}\int \hat{H}_2 \mathrm{d}t} Q_3\ |\psi_{L4}\rangle$, the mode $(0\ 1\ 0\ 0)^{\dagger}$ becomes occupied with non-vanishing amplitude, and can now contribute to the post-selection at the detector. The state of the measuring devices subject to the successful post-selection is $(1\ 0\ 0\ 0)Q_1\ |\psi_{L6}\rangle$, and is given by

$$|\psi_{AB}\rangle = -r^2|A\rangle|B\rangle + \frac{t^2}{4}(|A\rangle|B\rangle - |A'\rangle|B\rangle - |A\rangle|B'\rangle + |A'\rangle|B'\rangle). \tag{6}$$

The probabilities $P(k_1, l_2) = |\langle k_1|\langle l_2||\psi_{AB}\rangle|^2$ at leading non-vanishing order are given as

$$P(b_1, c_2) = P(c_1, b_2) = P(c_1, c_2) = \frac{r^4}{4}, \tag{7a}$$

$$P(b_1, b_2) = \frac{r^4}{4} + \epsilon_1\epsilon_2\frac{r^2t^2}{8}. \tag{7b}$$

The t^2 dependence in $P(b_1, b_2)$ is the evidence that a portion of the photon wave packet has been transmitted through the beam splitters BS_1 twice, and that the photon has been along the right arm of the outer interferometer MZ_1. We see that the quantum mechanical treatment of the system with two weak measurements in the two inner interferometers not only causes the photon to pass through the right arm of the outer interferometer, but it also leaves a trace that can be revealed by the atomic population of the meter atoms. This straightforward observation of ordinary quantum theory is in contradiction with the proposed theory of the past of the particle which does not account for the modification of the pre- and the post-selected states brought by the weak measurements [23]. If we apply the proposed theory to our scheme, then even after the two weak measurements, the forward and backward evolving states still overlap only along arm A, and the weak values of the photon number operator everywhere along the right arm are still *zero*. Thus according to the theory, the photon can not go through, and can not leave any observable trace along the right arm of MZ_1. This means that the sum of all the probabilities in equation (7) should be r^4. However, we clearly see that this is not the case. Oddly enough, the predictions of the proposed theory for the measurements are correct only if the measurements are not performed.

One can argue that the trace along the right arm of MZ_1 revealed by $P(b_1, b_2) \propto \epsilon_1\epsilon_2$ is second order contribution and need to be neglected in the proposed theory [23] based on TSVF. However, this means that TSVF, being a first order theory, should not be applied to the systems with multiple weak measurements, unless the disturbances caused by the weak measurements are properly taken into account [16–19]. This is an important point of this section and needs to be noted. This particular aspect of TSVF is generally not emphasized in the literature and the application of TSVF on the systems with multiple weak measurements results in controversies. For example, in the original paper [23] the claim that the photon was inside the inner interferometer but not on the path coming out of the inner interferometer is based on the first order theory and is not justified when two weak measurements are performed on the system. The second weak measurement is necessary to investigate the presence of the photon on the path coming out of the inner interferometer when it has been revealed inside the inner interferometer with the first weak measurement [24]. With two weak measurements in place, the application of TSVF on the system [23] becomes questionable.

4. Nested interferometer

In this section we put forward a different argument to refute the theory of the past of the particle [23]. In section 2 we saw that the prediction of the theory [23] for the presence of the photon inside the inner interferometer is correct only if the system is disturbed by actually performing a weak measurement on the system. The trick is that such measurement perturbs the destructive interference along the dark port of the inner interferometer, and a tiny amplitude leaks along the path to the detector [23–25]. However, it is not difficult to come up with a setup where a weak measurement, instead of disturbing destructive interference along a quantum channel, restores destructive interference along a channel that already had a tiny leakage. Thus the act of performing the weak measurement blocks the quantum channel between the place where the particle is revealed by the measurement and the detector. With this blocked channel in the way, standard quantum theory does not allow the given post-selection of the particle, and the theory [23] runs into problems.

Consider the setup shown in figure 3(a). The setup is similar to the one in figure 1(a), with the only difference being that there is an optical element placed along arm C of the inner interferometer, which introduces a slight phase change $i\phi$ on the amplitude passing through the arm. The destructive interference towards the detector (along arm F) is thus slightly disturbed and a tiny amplitude leaks out of the inner interferometer and reaches the detector D. The magnitude of this leakage is assumed to be of the same order as the one that results in [23, 30] when a weak measurement is performed inside the inner interferometer. The system in TSVF is shown in figure 3(b). The prediction of the theory of the past of the particle [23] for the presence of the particle inside the inner interferometer remains the same. However, in this setup, the prediction can run into problems with weak measurements. For example, we consider the weak measurement for the presence of the particle along the arm E of the inner interferometer. The weak measurement will slightly perturb the system and the amplitude along arm E will experience a slight phase shift $i\phi'$. This phase shift can compensate for the phase introduced by the optical element along arm C and can restore the destructive interference along the path F leading to the detector D. The standard quantum mechanical evolution of the system with the weak measurement at stage L3 is shown in table 4. At the stage L3 $i\phi$ is the phase introduced to the mode $(01\ 0)^\dagger$ by the optical element which is part of the setup, and $i\phi'$ is the phase introduced to the mode $(0\ 0\ 1)^\dagger$ by the weak measurement along arm E. For $\phi = \phi'$ the destructive interference for the mode $(0\ 1\ 0)^\dagger$ along path F (at stage L4) is restored. Standard quantum mechanics now clearly states that the particle revealed along arm E by the weak measurement cannot reach the detector and must leave the system at stage L4 along the channel $(0\ 0\ 1)^\dagger$. The theory [23] thus clearly runs into problems with the weak measurements which instead of slightly disturbing the destructive interference, restores the destructive interference.

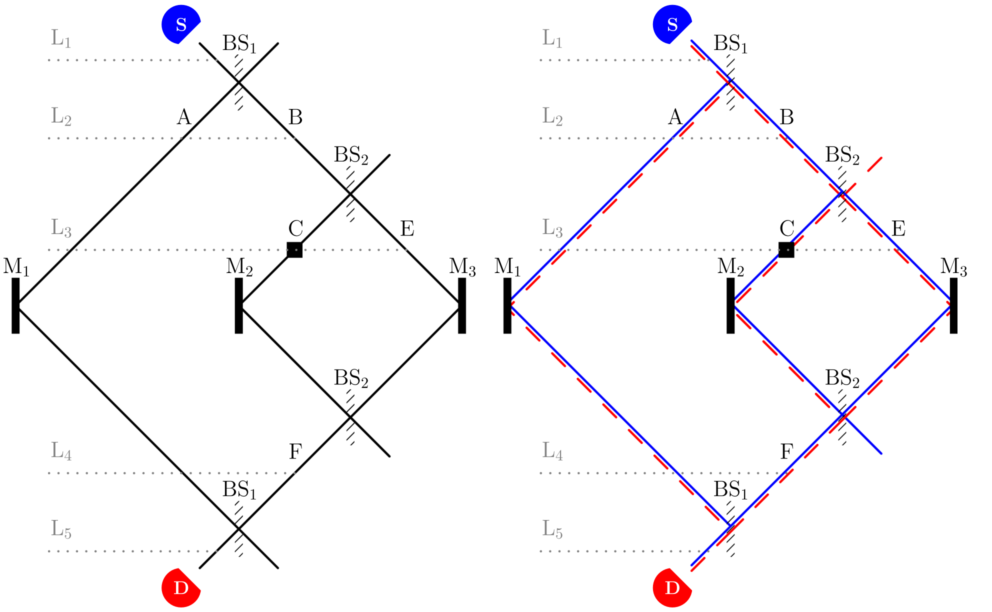

Figure 3. (a) System similar to figure 1(a) except for the difference that along arm C of the inner interferometer there is an optical element that introduces a small phase difference between the amplitudes along the two arms of the inner interferometer. (b) System in TSVF with the forward (solid blue) and the backward (dashed red) evolving states. Because of the optical element along the arm C, there is already a tiny leakage both for the forward and the backward evolving states along the arms F and B, respectively.

Table 4. Quantum evolution of the system with an optical element along the arm C of the inner interferometer that introduces a small phase $i\phi$ to the amplitude along this arm at stage L3. Additionally we perform a weak measurement along arm E that introduces the phase $i\phi'$ to the amplitude along this arm. For $\phi = \phi'$ the weak measurement along the arm E restores the destructive interference along the arm F along the mode $(0 \quad 1 \quad 0)^\dagger$ at stage L4. With such weak measurement the particle revealed along the arm E of the inner interferometer can not be post-selected at the detector D.

Stage	Quantum State $\lvert\psi\rangle$
L1	$(1 \quad 0 \quad 0)^\dagger$
L2	$-(\mathrm{i}r \quad -t \quad 0)^\dagger$
L3	$-\left(\mathrm{i}r \quad \mathrm{i}t\frac{1+\mathrm{i}\phi}{\sqrt{2}} \quad -t\frac{1+\mathrm{i}\phi'}{\sqrt{2}}\right)^\dagger$
L4	$-\left(\mathrm{i}r \quad \mathrm{i}t\frac{\phi'-\phi}{2} \quad \mathrm{i}t + t\frac{\phi+\phi'}{2}\right)^\dagger$
L5	$-\left(r\left(r + \mathrm{i}t\frac{\phi'-\phi}{2}\right) \quad rt\left(\mathrm{i} + \frac{\phi'-\phi}{2}\right) \quad t\left(\mathrm{i} + \frac{\phi+\phi'}{2}\right)\right)^\dagger$

5. Conclusion

We have considered quantum mechanical and TSVF descriptions of a couple of nested Mach–Zehnder interferometer schemes that make use of destructive interference. It is shown that a particular interpretation of TSVF that proposes a theory for the past of a quantum particle runs into problems when weak measurements are performed to probe the past. The problems stem from the assumption in the theory that weak measurements do not significantly modify the quantum system. In fact, any disturbance to destructive interference is always significant. Any small amplitude created by the action of the weak measurement can combine with the amplitudes along other quantum channels and give rise to observable effects. This quantum trick has been used to propose the theory for the past of the particle. We show that the same trick can refute the theory. More generally, we show that care must be taken in applying TSVF to describe quantum systems that make use of delicate interference effects. The quantification of weak measurements and interpretation of weak values in such systems is an open question.

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